# Homework 6 for Math 480A

## http://wiki.wstein.org/2008/480a

## Due Wednesday May 14, 2008

Each problem has equal weight, and parts of problems are worth the same amount as each other. There are **5 problems**. I have office hours MWF 2:30-3:30 in Sieg 312, unless otherwise stated. You can email me about problems; all responses will be cc'd to `sage-uw`, so you may want to subscribe to that mailing list.

You may have to read up on some of the basic definitions in graph theory, in particular the notions of chromatic polynomial, adjacency matrix, eigenvalues, planarity, and automorphism group. Looking at some random web pages might help.

1. Compute the chromatic polynomial of the $n$-cube graph for $n = 2, 3, 4$. [Hint: the Sage command to make that graph is `graphs.CubeGraph(n)`.]

2. (a) Compute all the eigenvalues (*not* eigenvectors) of the adjaceny matrix of the $n$-cube graph for $n = 2, 3, 4, 5$.

   (b) Make a conjecture about these eigenvalues.

3. List all subsets of the set $\{2, 3, 5, 7, 11\}$ of prime numbers up to 11.

4. Consider the set of integers $\{1, 2, 3, \ldots, 15\}$. View these as the vertices of a graph $G$ that has an edge between two vertices $n$ and $m$ if and only if the difference $n - m$ is a positive prime number.

   (a) Define $G$ in Sage and draw a picture of $G$.

   (b) How many connected components does $G$ have?

   (c) Is $G$ planar?

   (d) What is the automorphism group of $G$?

5. (a) Use Sage to enumerate all nonplanar graphs, up to isomorphism, on 6 vertices. [Hint: The function `graphs(n)` in Sage returns an iterator over all graphs on $n$ vertices.]

   (b) Draw a picture that illustrates all the graphs found in part (a).