Homework 5 for Math 480A

http://wiki.wstein.org/2008/480a

Due Wednesday May 7, 2008

Each problem has equal weight, and parts of problems are worth the same amount as each other. There are **4 problems**. I have office hours MWF 2:30-3:30 in Sieg 312, unless otherwise stated. You can email me about problems; all responses will be cc'd to `sage-uw`, so you may want to subscribe to that mailing list.

1. Let $p_1, p_2, p_3, \ldots$ be the sequence of primes numbers. Create the 10×10 matrix A with prime integer entries p_1, p_2, p_3 and so on.

$$A = \begin{pmatrix} 2 & 3 & 5 & 7 & 11 & 13 & 17 & 19 & 23 & 29 \\ 31 & 37 & 41 & 43 & 47 & 53 & 59 & 61 & 67 & 71 \\ 73 & 79 & 83 & 89 & 97 & 101 & 103 & 107 & 109 & 113 \\ 127 & 131 & 137 & 139 & 149 & 151 & 157 & 163 & 167 & 173 \\ 179 & 181 & 191 & 193 & 197 & 199 & 211 & 223 & 227 & 229 \\ 233 & 239 & 241 & 251 & 257 & 263 & 269 & 271 & 277 & 281 \\ 283 & 293 & 307 & 311 & 313 & 317 & 331 & 337 & 347 & 349 \\ 353 & 359 & 367 & 373 & 379 & 383 & 389 & 397 & 401 & 409 \\ 419 & 421 & 431 & 433 & 439 & 443 & 449 & 457 & 461 & 463 \\ 467 & 479 & 487 & 491 & 499 & 503 & 509 & 521 & 523 & 541 \end{pmatrix}$$

   Let v be the vector with entries the first 10 primes $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$.

   (a) What is the least common multiple of the denominators of the entries of the unique vector v with $Ax = v$.

   (b) Do as above but with 10 replaced by 100, i.e., A is a 100×100 matrix.

2. Describe your final project in more detail and outline what you will do. Write this up clearly enough that a random student in the class will understand what you are proposing. The student who is grading your homework should comment on whether the proposed project is described clearly, is interesting to them, etc.

3. There are 8 ordered lists of positive integers that sum to 4:

```
[1, 1, 1, 1]
[1, 1, 2]
[1, 2, 1]
[1, 3]
[2, 1, 1]
[2, 2]
[3, 1]
[4]
```

   How many ordered lists of integers sum to 2008?

4. (a) Compute the sum s of the 100th powers of the integers up to 10^7 (ten million). For your answer just give the number of digits of s (don't write out s itself).

   (b) Evaluate the infinite sum

$$\sum_{n=1}^{\infty} \frac{1}{n^{100}}$$

   of the inverses of the hundreth powers of positive integers.