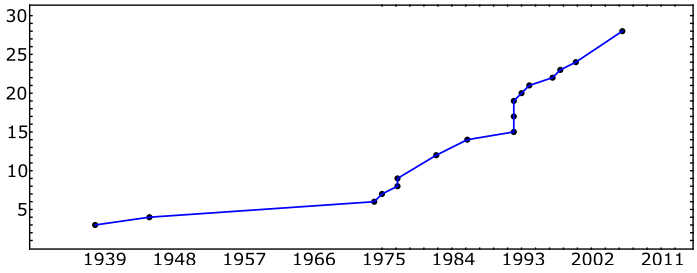
Exercises for Part 2, Section 1: Computational Problems Associated to the BSD Conjecture

Math 582e, Winter 2009, University of Washington

Due Wednesday February 25, 2009

1. Estimate whether or not you will live to see an elliptic curve over $\mathbb{Q}$ of rank 100. Use the internet to find out when people first found curves of each rank, then interpolate based on some model you make up.



2. Let E be the elliptic curve defined by $y^2 + xy + 3y = x^3 + 2x^2 + 4x + 5$.
   (a) Compute all the numbers $b_2, b_4, b_6, b_8, c_4, c_6, \Delta, j$ associated to the Weierstrass equation
   (b) Use Sage to compute generators for the Mordell-Weil group $E(\mathbb{Q})$.

3. Let E be the elliptic curve defined by $y^2 = x^3 - 16x + 16$. Find 5 square-free integers $D < -4$ such that $\mathrm{ord}_{s=1} L(E^D, s) = 0$ and 37 splits in the quadratic field $\mathbb{Q}(\sqrt{D})$.