The Story

Mazur @ Perci Conference

Theorem.

Conjecture. —— guess / heuristic support / data support

Expectation. —— guess / " / data going contrary

(CONFLICT)

raisin in pudding

Quartic Fields ordered by increasing discriminant

$K = \mathbb{Q}(\theta)$

4 / $\mathbb{Q}$

θ satisfies $X^4 + aX^3 + bX^2 + cX + d = 0$.

Manjul Bhargava's

Cautionary Tale

$$\lim_{X\to\infty} \frac{\#\{\text{quartic fields, } -\text{totally real} -\text{disc} \in [0,X]\}}{X}$$

$$= \frac{1}{48} \prod_{p \text{ primes}} \frac{1}{(1+p^{-2}-p^{-3}-p^{-4})}$$

90.644...% Galois group S_4

9.356% —— D_4

0 % —— all others.

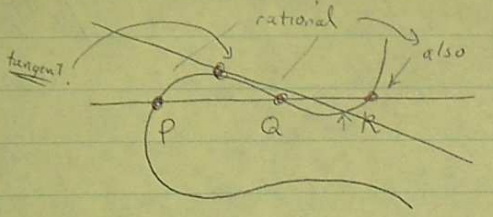
Todo: Draw the graph for this and see how it converges to the answer.

Elliptic Curves:

$y^2 = x^3 + ax + b$ $\quad a, b \in \mathbb{Q}$

x^3+ax+b no multiple roots.

rational point: (x_0, y_0) — solve the equation.



chord and tangent process

[slides]

Type: 0 no rational points

1 all from one via chord and tangent (mod not torsion)

basically rank, but through in torsion: 2 — 2 — not 1

⋮

r — r

conj ⇒ same as rank for questions

Check this – draw graphs for this

Parity Conjecture: - even (rank) types — 50%
- odd types — 50%

(not what I would call that! – William)

Minimalist Philosophy:

type 0 – 50%

type 1 – 50%

type ≥2 – 0%.

??
oo

Mazur

- all elliptic curves
- square free conductor
- prime conductor
- twists of a given elliptic curve. E $\quad y^2 = x^3 + ax + b$

$dy^2 = x^3 + ax + b$

varying d (square free)

Conj (1979)
Goldfeld
minimalist philosophy
type 0: 50%
type 1: 50%

%-age types of ranks — quadratic twist

Katz-Sarnak:

Elementary Heuristic

$\#\{$elliptic curves — quad twist family $- |d| \leq X\}$ with rank even and ≥ 2, $\sim X^{3/4+\varepsilon}$ $\quad X^{3/4-\varepsilon}$

$x^3 + y^3 = d$

varying d (cube free)

Random Matrix heuristic:

$\sim c \cdot X^{3/4} \cdot (\log X)^b$

minimalist philosophy.