

Introduction
to Sage

William Stein

History and
Goals

A Demo

A Short Introduction to Sage

William Stein



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The Sage Project

Mission Statement

Create a viable free open source alternative to Magma, Maple, Mathematica, and Matlab

A “viable alternative” will have...

- The *mathematical features* of Magma, Maple, Mathematica, and Matlab with *comparable speed*.
- Beautiful interactive 2d and 3d graphics.
- A notebook interface and an IDE.
- Many books (full undergraduate curriculum)
- A web application interface.

Sage ain't Octave (=open source MATLAB clone)

Sage need *not* run programs written in the custom math-only languages of Magma, Mathematica, etc.

SAGE

Software for Arithmetic Geometry Experimentation

- I needed an *open source* alternative to Magma. David Joyner (coding theorist) had similar concerns.
- SAGE in 2005 – number theory (PARI) and coding theory (GAP) – no symbolic calculus or numerical computation.

Number theory & Coding theory: started out very technical

```
sage: E = EllipticCurve('389a'); E
Elliptic Curve defined by y^2 + y = x^3 + x^2 - 2*x
sage: E.gens()
[(-1 : 1 : 1), (0 : -1 : 1)]
sage: G = matrix(GF(5), 4, 7, [1,1,1,0,0,0,0,1,0,0,1,1,...
sage: C = LinearCode(G); C
Linear code of length 7, dimension 4 over Finite Field ...
sage: C.minimum_distance()
3
```

Why not Magma?



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- 1 **Commercial:** Expensive for my collaborators and students (“third world discount” = 3 months salary)
- 2 **Closed:** Implementation of algorithms often secret
- 3 **Frustrating:** Too tight control of development
- 4 **Static:** Users can’t define their own classes (data types)
- 5 **Copy protection:** A pain in the arse
- 6 **Language:** No eval, no exception handling, no namespaces, little development of math-only language
- 7 **Developer community:** too small, no public mailing list
- 8 **Graphics:** No graphics, symbolic calculus, or GUI
- 9 **Bugs:** No public bug tracker or list of reported bugs
- 10 **Compiler:** No compiler (nothing like Cython)

(Related remarks for Maple, Mathematica, and MATLAB.)



What is Sage?

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- ① **A self-contained distribution** of over 90 open source packages that is easy to build from source.
- ② **Interfaces** that smoothly tie together all these libraries and packages.
- ③ **A new library** that implements novel algorithms. About a half million lines of code written by a worldwide community of over 200 people over the last 5 years.
<http://sagemath.org/development-map.html>

Demo

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A Demo...

Use Sage From the Command Line

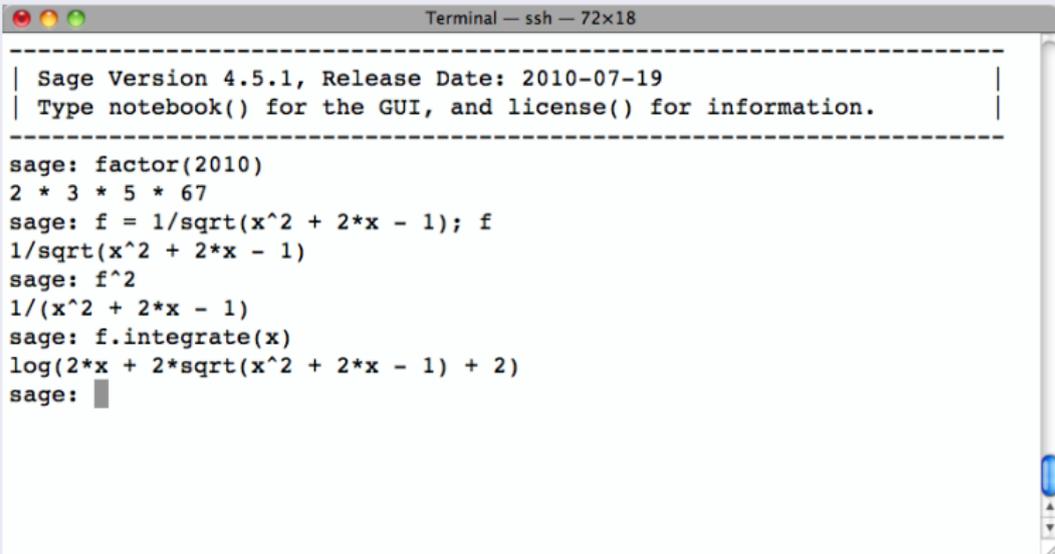
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Command Line Sage



```
Terminal — ssh — 72x18
-----
| Sage Version 4.5.1, Release Date: 2010-07-19 |
| Type notebook() for the GUI, and license() for information. |
-----

sage: factor(2010)
2 * 3 * 5 * 67
sage: f = 1/sqrt(x^2 + 2*x - 1); f
1/sqrt(x^2 + 2*x - 1)
sage: f^2
1/(x^2 + 2*x - 1)
sage: f.integrate(x)
log(2*x + 2*sqrt(x^2 + 2*x - 1) + 2)
sage: █
```

Use Sage Via the Notebook

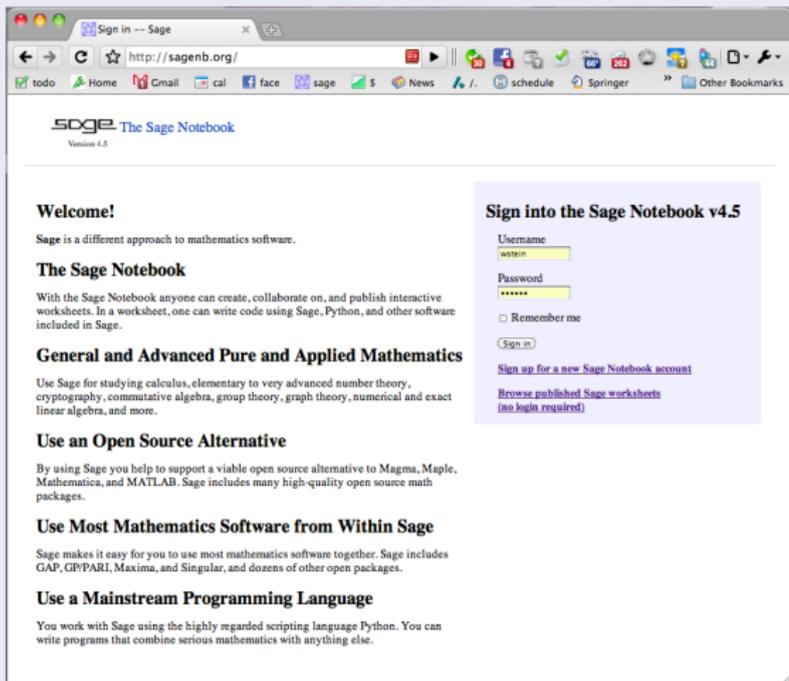
Web Browser Based Interface to Sage

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The screenshot shows a web browser window with the address bar displaying `http://sagenb.org/`. The page title is "Sign in -- Sage". The main content area features the Sage logo and the text "The Sage Notebook Version 4.5". Below this, there is a "Welcome!" section followed by a paragraph: "Sage is a different approach to mathematics software." The "The Sage Notebook" section explains that users can create, collaborate on, and publish interactive worksheets. The "General and Advanced Pure and Applied Mathematics" section lists various mathematical fields supported by Sage. The "Use an Open Source Alternative" section mentions Magma, Maple, Mathematica, and MATLAB. The "Use Most Mathematics Software from Within Sage" section lists GAP, GP/PARI, Maxima, and Singular. The "Use a Mainstream Programming Language" section notes that Sage uses Python. On the right side, there is a "Sign into the Sage Notebook v4.5" form with fields for "Username" (containing "wstein") and "Password" (masked with dots), a "Remember me" checkbox, and a "Sign in" button. Below the form are links for "Sign up for a new Sage Notebook account" and "Browse published Sage worksheets (no login required)".

Demo: Factoring

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Factoring an integer:

```
factor(2012)
```

$$2^2 \cdot 503$$

Factoring a symbolic expression:

```
x,y=var('x,y'); factor(x^3 - sin(y)^3)
```

$$(x - \sin(y))(x^2 + x \sin(y) + \sin(y)^2)$$

Factoring a polynomial over a nontrivial finite field:

```
F.<alpha> = GF(49); x = polygen(F)  
factor(x^4 + x^3 - 2)
```

$$(x + \alpha + 1) \cdot (x + 6\alpha + 2) \cdot (x + 6)^2$$

Demo: Solving Equations

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Solve a quadratic equation:

```
x = var('x'); solve(x^2 + 7*x == 5, x)[0]
```

$$x = -\frac{1}{2} \sqrt{69} - \frac{7}{2}$$

Solve a system of two linear equations with one unknown coefficient α :

```
var('alpha, y')  
solve([3*x + 7*y == 2, alpha*x + 3*y == 8],  
x, y)
```

$$\left[\left[x = \frac{50}{7\alpha-9}, y = \frac{2(\alpha-12)}{7\alpha-9} \right] \right]$$

Demo: Computing Symbolic Integrals

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```
f = 1/sqrt(x^2 + 2*x - 1); f.integrate(x)
```

$$\log\left(2x + 2\sqrt{x^2 + 2x - 1} + 2\right)$$

```
g = integrate(sin(x)*tan(x), x); g
```

$$-\frac{1}{2} \log(\sin(x) - 1) + \frac{1}{2} \log(\sin(x) + 1) - \sin(x)$$

```
h = g.diff(x); h
```

$$\frac{-\cos(x)}{2(\sin(x)-1)} + \frac{\cos(x)}{2(\sin(x)+1)} - \cos(x)$$

```
(h - sin(x)*tan(x)).simplify_full()
```

0

Demo: Plotting a 2D Function

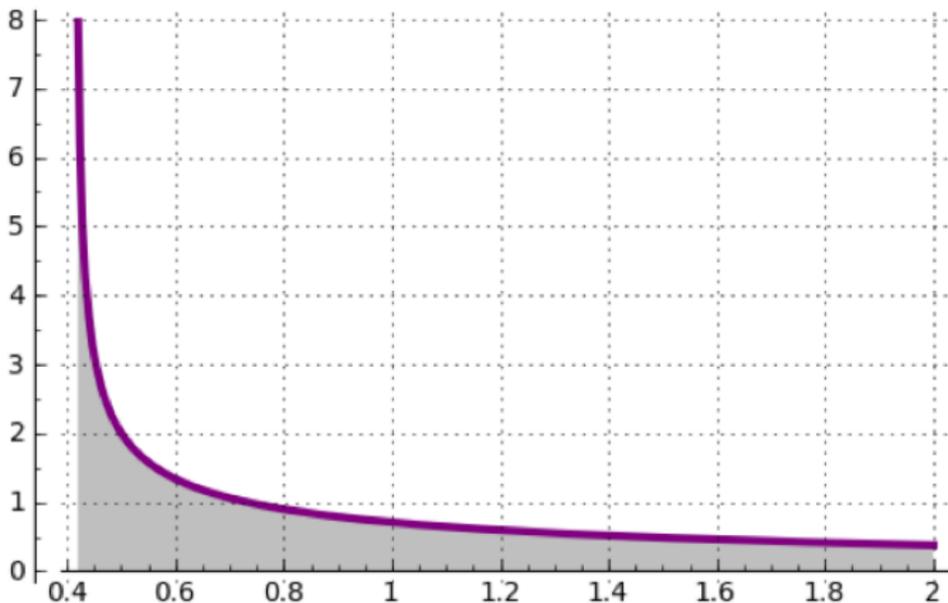
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```
plot(1/sqrt(x^2 + 2*x - 1), (x,.4,2), thickness=3,  
     color='purple', fill=True, gridlines=True)
```



Demo: Plotting a 3D Function

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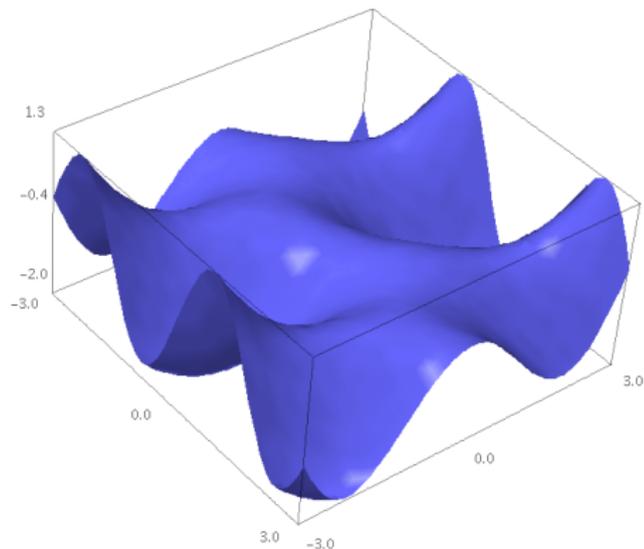
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```
var('x,y');plot3d(sin(x-y)*y*cos(x),(x,-3,3),(y,-3,3))
```

(x,y)



Demo: Interactive image compression

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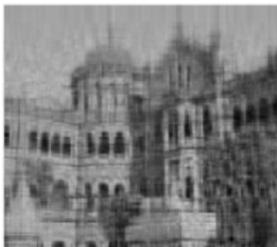
```
import pylab; import numpy
A_image = numpy.mean(pylab.imread(DATA + 'mumbai.png'), 2)
u,s,v = numpy.linalg.svd(A_image)
S = numpy.zeros( A_image.shape )
S[:len(s),:len(s)] = numpy.diag(s)
n = A_image.shape[0]
@interact
def svd_image(i = ("Eigenvalues (quality)",(20,(1..A_image.shape[0])))):
    A_approx = numpy.dot(numpy.dot(u[:, :i], S[:i, :i]), v[:i, :])
    g = graphics_array([matrix_plot(A_approx), matrix_plot(A_image)])
    show(g, axes=False, figsize=(8,3))
    html("Compressed to %.1f%% of size using %s eigenvalues."%(
        100*(2.0*i*n+i)/(n*n), i))
```

Eigenvalues (quality)



20

Compressed to 12.5% of size using 20 eigenvalues.



Website Tour

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Quick Tour of Website

Questions

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Questions?