Some Curves With Small Regulator

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I examined Cremona's newest data for elliptic curves of conductor ≤ 12000. The four smallest regulator curves in his tables are as follows:

Curve	Regulator = Min height of a point
3990V1	0.0089143244556863533937
3630Y1	0.0090386818898339096889
1430K1	0.0097398661429109179437
1470L1	0.0099641079999126058232

These curves all have rank 1. Here is some more information about each:

N	$[a_1, a_2, a_3, a_4, a_6]$	Point	N	Δ
3990	$[1, 1, 1, -125615, 61201397]$	$(7107, -602054)$	$2 \cdot 3 \cdot 5 \cdot 7 \cdot 19$	$2^{11} \cdot 3^4 \cdot 5^7 \cdot 7^5 \cdot 19^3$
3630	$[1, 0, 0, -141875, 18393057]$	$(-386, 4153)$	$2 \cdot 3 \cdot 5 \cdot 11^2$	$2^{17} \cdot 3^5 \cdot 5^7 \cdot 11^4$
1430	$[0, 0, 1, -1089, 14308]$	$(11, 60)$	$2 \cdot 5 \cdot 11 \cdot 13$	$2^{13} \cdot 5^5 \cdot 11^3 \cdot 13^2$
1470	$[1, 1, 1, -2990, 71147]$	$(27, -119)$	$2 \cdot 3 \cdot 5 \cdot 7^2$	$2^{13} \cdot 3^2 \cdot 5^5 \cdot 7^4$

The point $(7107, -602054)$ on **3990V1** is entertaining because its naive and canonical heights differ by almost 9. Watch $2^n P$ get small then become large.

```
> E := EC("3990V1");
> P := E![7107,-602054];
> 2*P;
(1787 : -75374 : 1)
> 4*P;
(457 : -10204 : 1)
> 8*P;
(-2537/9 : -228958/27 : 1)
> 16*P;
(5443/9 : 380182/27 : 1)
> 32*P;
(-3185573/47961 : 87754320614/10503459 : 1)
> 64*P;
(7496507080486748587/42278403738202641 :
      -58793256118747632513363890206/8693163995352094662637689 : 1)
```