NOTATION

As usual $\mathbf{Z}$ will denote the ring of integers, $\mathbf{Q}$ will denote the field of rational numbers, $\mathbf{C}$ the field of complex numbers, $\mathbf{Z}_2$ the ring of 2-adic integers and $\mathbf{Q}_2$ the field of 2-adic numbers. Of course, $\mathbf{Q}$ is canonically embedded as a subfield of both $\mathbf{C}$ and of $\mathbf{Q}_2$. The field $\mathbf{Q}_2$ is endowed with its canonical discrete valuation, which we denote ord_2, normalized so that $\mathrm{ord}_2(2) = 1$.

We let $\overline{\mathbf{Q}}$ denote the algebraic closure of $\mathbf{Q}$ in $\mathbf{C}$. We choose an algebraic closure $\overline{\mathbf{Q}}_2$ of the field $\mathbf{Q}_2$ of 2-adic numbers, and fix an embedding of $\overline{\mathbf{Q}}$ into $\mathbf{Q}_2$, which identifies $\overline{\mathbf{Q}}$ with the algebraic closure of $\mathbf{Q}$ in $\overline{\mathbf{Q}}_2$.

The valuation ord_2 extends canonically to a (non-discrete) valuation on $\overline{\mathbf{Q}}_2$, which we continue to denote by ord_2. We may complete $\overline{\mathbf{Q}}_2$ with respect to this valuation to obtain a complete algebraically closed field which we denote by $\mathbf{C}_2$. The valuation on $\overline{\mathbf{Q}}_2$ extends canonically to a valuation on $\mathbf{C}_2$, which we continue to denote by ord_2. We write $\mathcal{O}_2$ for the valuation ring of this valuation.

We define an absolute value on the field $\mathbf{C}_2$ as follows: if $r \in \mathbf{C}_2$, then

$$|r| = 2^{-\mathrm{ord}_2(r)}.$$

The ring $\mathcal{O}_2$ becomes identified with closed unit disc around the origin in $\mathbf{C}_2$, while the maximal ideal of $\mathcal{O}_2$ becomes identified with the open unit disc around the origin.

For us, a ring will always be assumed to be commutative, with an identity.

If R is a topological ring, and $t_1, \cdots, t_m$ and $x_1, \cdots, x_n$ are two collection of variables then

$$R[[t_1, \ldots, t_m]]\langle x_1, \ldots, x_n\rangle$$

denotes the subring of the power series ring $R[[t_1, \ldots, t_m, x_1, \cdots, x_n]]$ consisting of those power series

$$\sum_{i_1,\ldots,i_m,j_1,\ldots,j_n} r_{i_1,\ldots,i_m,j_1,\ldots,j_n} t_1^{i_1} \cdots t_m^{i_m} x_1^{j_1} \cdots x_n^{j_n} \in R[[t_1, \ldots, t_m, x_1, \ldots, x_n]]$$

such that, for each fixed multi-index $i_1, \ldots, i_m$,

$$\lim_{j_1+\cdots+j_n \to \infty} r_{i_1,\ldots,i_n,j_1,\ldots,j_n} = 0.$$

In the particular case when $R = \mathbf{Z}_2$, this is the ring of rigid analytic functions of norm less than or equal to one on the product of m copies of the open unit disc and n copies of the closed unit disc.

We also use the standard abuses of this notation which occur in rigid analysis, in which some relations are allowed between the various t_i and x_i. For example, $\mathbf{Z}_2[[u]]\langle 64/u\rangle$ is used to denote the ring $\mathbf{Z}_2[[u]]\langle v\rangle/(uv - 64)$.