MR1330929 (96c:11058) 11G05 (14H52)

Kamienny, S. (1-SCA); **Mazur, B. [Mazur, Barry C.]** (1-HRV)

Rational torsion of prime order in elliptic curves over number fields. (English summary)

With an appendix by A. Granville.

Columbia University Number Theory Seminar (New York, 1992).

Astérisque No. 228 (1995), 3, 81–100.

For any integer $d \geq 1$, let $S(d)$ be the set of primes p such that there exists a number field K of degree d and an elliptic curve E/K such that $E(K)$ contains a point of order p. In his famous Eisenstein ideal paper, Mazur [Inst. Hautes Études Sci. Publ. Math. No. 47 (1977), 33–186 (1978); MR0488287 (80c:14015)] proved that $S(1) = \{2, 3, 5, 7\}$, and Kamienny [Invent. Math. **109** (1992), no. 2, 221–229; MR1172689 (93h:11054)] extended Mazur's methods and used ideas concerning embeddings of symmetric products into modular Jacobian varieties to prove that $S(2) = \{2, 3, 5, 7, 11, 13\}$. In this paper the authors show that for all d, the set $S(d)$ has (natural) density zero. Recently, L. Merel ["Bornes pour la torsion des courbes elliptiques sur les corps de nombres", to appear] has extended the methods of Mazur and Kamienny to prove the full boundedness conjecture that $S(d)$ is finite for all d. Thus the results of the present paper have been largely superseded, although the methods are still of interest, as are some of the questions raised by the authors.

{See also the following review.}

{For the entire collection see MR1330924 (95m:11006)}

Reviewed by *Joseph H. Silverman*