Budget Justification Page

A SALARY:
B OTHER PERSONNEL:
C FRINGE BENEFITS:
D EQUIPMENT:

1. Computer: A Dell Server with 48 cores and 256GB RAM: **$25,526**

E TRAVEL:
F OTHER DIRECT COSTS:

1. **Student Salary:** The PI is requesting salary for 1 graduate student for 1 hour per week at $20/hour for 3 years: $1 \times 52 \times 20 \times 3 = \$$**3120**

   The student will do routine systems administration of the computer (upgrades, backups, creation of accounts, etc.).

G MATERIALS AND SUPPLIES:
I INDIRECT COSTS:

1 Budget justification

The collaborative nature of the project requires advanced software development tools (ticket server, distributed version control), as well as heavy computational resources (multi-platform compilation, regression testing, benchmarking; see § 1.1). So far, the Sage-Combinat project has been using a virtual server, courtesy of the Sage team at the University of Washington (`combinat.sagemath.org`). Scaling further will require its replacement by a modern server, to be hosted as part of the Sage computation farm (which currently includes four GNU/Linux servers with 24 cores and 128Gb of RAM). It will be accessible remotely for all participants, through ssh or the Sage notebook interface. This server will also be used for time or memory demanding calculations (see § 1.2). Unused resources will be made available to the Sage-Combinat and Sage community at large and will continue to be a resource for the Sage-Combinat community well after the end of this project.

An important artifact is that our computing needs, both for software development and computer exploration, usually come in bursts. Beside, assuming appropriate software preconfiguration (which rules out using typically available supercomputers which are preconfigured for "scientific computing"), running such large calculations remotely is easy. It is therefore a much better investment (both from a financial and system administration point of view) to share a single large computing server, as opposed to several smaller ones hosted at local institutions, as it spreads the load and thus guarantees a continuous use of the computing power.

We chose the quote from Dell over the ones from Oracle and Silicon Mechanics, because Oracle's was too high, and though Silicon Mechanics quote was a little lower, Dell is a much larger vendor with an excellent relationship with the University of Washington mathematics department.

1.1 Software development needs

Sage is a rather large and complex piece of software. Its stability relies very much on its review process. Lately, this review process, together with the subsequent release management, has become a bottleneck in Sage's development. This is being tackled by the Sage developers by a progressive automation of the workflow. This automation is particularly vital for a project mostly run by volunteers whose main job is research, even if the price is a heavy use of computing power.

Let us give some figures. The Sage-Combinat queue continuously holds around 200 patches; at some Sage releases more than 30 of them were merged at once. One important part of the review and release process is the running of Sage's regression test, which ensures that new features and bug fixes do not introduce problems; a typical patch review requires running the tests a dozen times. Running them on an average laptop takes two hours, compared to a couple minutes on the current Sage's multicore server, and tentatively one on the requested server. This difference totally conditions the feasibility of reviews.

Beside, the current Sage servers are being progressively saturated, and the Sage-Combinat community is growing at a fast pace.

1.2 Outstanding computer exploration

All the recent results of the participants made use of heavy computer exploration at some point. Due to the typical combinatorial explosion in algebraic combinatorics, the involved calculations can just not be run on a desktop computer nor even on an average computation server. In the following, we give three outstanding examples of calculations that would be made possible by the requested server.

1.2.1 Classification of finite affine crystals

Kashiwara conjectured [Kas05, Introduction] that any 'good' finite affine crystal is an n-fold tensor product of Kirillov-Reshetikhin crystals (see [Kas02, Section 8] for a definition of 'good') for some n. This would result in a classification of these crystals. In [BST10] we proved this result in type A and $n = 2$ for crystals coming from promotion operators, and found further evidence for this conjecture for $n > 2$ using heavy computer exploration with MuPAD-Combinat involving crystal graph isomorphisms. This also revealed new structures about these crystals.

Pursuing computer exploration for larger n would allow to better understand, and make use of, the symmetries, which is the main bottleneck for tackling $n > 2$. This would be made possible by the combination of the requested many-core server, of Sage's parallel features and of a parallel implementation of the crystal isomorphism algorithmic, in order to tackle the exponential growth with n of the exploration space.

1.2.2 Combinatorial conjectures on Bruhat order

There are a number of combinatorial conjectures in Bump and Nakasuji [BN10] that have been tested using Sage for simply-laced Cartan types, but only up to A_5 and D_4, by lack of a powerful enough computer. Pushing those computations further is necessary to gain confidence in the conjectures.

1.2.3 Computation with Macdonald polynomials

In the paper where k-Schur functions were introduced [LLM03], it was conjectured that an appropriate subset of Macdonald polynomials expand positively in the basis of k-Schur functions. This conjecture is still open, and has only been verified to a relatively low degree (around $n = 15$). This is due to the difficulty of computing large Macdonald polynomials. A publicly available, easily accessible table of precomputed Macdonald polynomials would simplify exploration of this and other similar conjectures.

References

[BN10] Daniel Bump and Maki Nakasuji. Integration on p-adic groups and crystal bases. *Proc. Amer. Math. Soc.*, 138(5):1595–1605, 2010.

[BST10] Jason Bandlow, Anne Schilling, and Nicolas M. Thiry. On the uniqueness of promotion operators on tensor products of type a crystals. *Journal of Algebraic Combinatorics*, 31, Mai 2010. arXiv:0806.3131 [math.CO].

[Kas02] Masaki Kashiwara. On level-zero representations of quantized affine algebras. *Duke Math. J.*, 112(1):117–175, 2002.

[Kas05] Masaki Kashiwara. Level zero fundamental representations over quantized affine algebras and Demazure modules. *Publ. Res. Inst. Math. Sci.*, 41(1):223–250, 2005.

[LLM03] L. Lapointe, A. Lascoux, and J. Morse. Tableau atoms and a new Macdonald positivity conjecture. *Duke Math. J.*, 116(1):103–146, 2003.