Mathematics 21b.
Linear Algebra and Differential Equations

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EIGENAPPLICATIONS!!!

Theorem 1 (Cayley-Hamilton). *Let A be an $n \times n$ matrix, and let $p(t) = t^n + c_{n-1}t^{n-1} + \cdots + c_1 t + c_0$ be its characteristic polynomial. Then, $p(A) = A^n + c_{n-1}A^{n-1} + \cdots + c_1 A + c_0 I_n = 0$.*

In other words, when you plug a matrix into its own characteristic polynomial, and treat the resulting expression as a linear transformation, you find that it is indeed the zero transformation! Hence, a matrix satisfies its own characteristic polynomial! Let's spend some time now proving it in certain cases. They are set as exercises, and can be found on the following pages.

Exercise 1. *Prove the Cayley-Hamilton Theorem for an arbitrary* 2×2 *matrix.*

Exercise 2. *Prove the Cayley-Hamilton Theorem for an arbitrary* $n \times n$ *diagonal matrix.*

Exercise 3. *Prove the Cayley-Hamilton Theorem for an arbitrary diagonalisable $n \times n$ matrix.*

EXAM REMINDERS:

The second midterm exam is a week from Monday! It will undoubtedly be more challenging than the first. In preparation, therefore, you should be doing and taking note of the following:

- Start downloading practice exams from the website and doing them!
- Do practice true/false questions!
- Talk to your TF's and CA's about any concepts with which you are having difficulty—remember, we are here to help you! Go to office hours! Go to section!
- **I will conduct a review session on Saturday, 13 April from 11:30 AM - 2:30 PM in Science Center Hall C.** It is helpful if you email me the questions you would like me to answer or the concepts you would like me to review *a day in advance.* This gives me time to prepare.
- **I have office hours on Monday, 8 April from 8:00 PM - 10:00 PM in Loker Commons.**

GOOD LUCK!!!—or, as we say in the mathematics business, "May all your matrices be diagonalisable!"