

Exercise Set 3:
Integers Modulo n

Math 414, Winter 2010, University of Washington

Due Wednesday, January 27, 2010

1. Let n be a positive integer and let

$$P = \{a : 1 \leq a \leq n \text{ and } \gcd(a, n) = 1\}.$$

Is it necessarily the case that

$$\prod_{a \in P} a \equiv -1 \pmod{n}?$$

2. (a) Find an integer x such that

$$x \equiv 3 \pmod{7} \quad \text{and} \quad x \equiv 5 \pmod{11}.$$

- (b) Find an integer x such that

$$x \equiv -1 \pmod{2010} \quad \text{and} \quad x \equiv 1 \pmod{2011}.$$

3. Find all *four* solutions to the equation

$$x^2 - 1 \equiv 0 \pmod{100}.$$

4. Suppose that $n > 1$ is an integer and that $2^{n-1} \equiv -1 \pmod{n}$. Is it possible that n is prime?
5. Find an integer x such that $5x + 7 \equiv 2010 \pmod{2011}$.