Math 480 (Spring 2007): Homework 7

Due: Monday, May 14

There are 6 exciting problems. Each problem is worth 6 points and parts of multipart problems are worth equal amounts. You may work with other people and use a computer, unless otherwise stated. Acknowledge those who help you.

1. Find a continued fraction that equals each of the following rational numbers:
   (a) $13/7$
   (b) $-9/13$
   (c) $21/13$
2. Find the value (which is a rational number) of each of the following continued fractions.
   (a) $[1, 2, 3]$
   (b) $[0, 1, 5, 2]$
   (c) $[3, 7, 15]$
3. Let f_n be the nth Fibonacci number, so $f_1 = 1$, $f_2 = 1$, and for $n \geq 3$ we have $f_n = f_{n-1} + f_{n-2}$. Prove that the continued fraction expansion of f_{n+1}/f_n consists of n 1's, i.e.,
   $$\frac{f_{n+1}}{f_n} = [1, 1, \ldots, 1].$$
4. Prove that if $[a_0 \ldots, a_n]$ and $[b_0, \ldots b_m]$ are two simple continued fractions that have the same value, and that $a_i > 0, b_j > 0$ for all i, j, and $a_n > 1$ and $b_m > 1$, then $n = m$ and $a_i = b_i$ for all i. Thus the continued fraction expansion of a rational number is unique if the last term is required to be larger than 1.
5. Show how to use continued fractions to find a rational number a/b in lowest terms such that
   $$\left|\frac{a}{b} - \sqrt[3]{2}\right| < \frac{1}{b^2} < 0.001.$$
6. The number 0.195876 is a decimal approximation to a rational number a/b with $|b| < 100$. Show how to use continued fractions to find a/b.