Math 129: Algebraic Number Theory
Homework Assignment 8

William Stein

Due: Thursday, April 15, 2004

1. Let k be any field. Prove that the only nontrivial valuations on $k(t)$ which are trivial on k are equivalent to the valuation (3.3) or (3.4) of Lecture 16.

2. A field with the topology induced by a valuation is a topological field, i.e., the operations sum, product, and reciprocal are continuous.

3. Give an example of a non-archimedean valuation on a field that is not discrete.

4. Prove that the field $\mathbf{Q}_p$ of p-adic numbers is uncountable.

5. Prove that the polynomial $f(x) = x^3 - 3x^2 + 2x + 5$ has all its roots in $\mathbf{Q}_5$, and find the 5-adic valuations of each of these roots. (You might need to use Hensel's lemma, which we haven't discussed in class. See Appendix C of Cassels, which you may reference.)