Homework 6 for Math 581F
Due FRIDAY November 9, 2007

Each problem has equal weight, and parts of problems are worth the same amount as each other. This homework assignment is short because of the midterm this weekend.

1. (a) Give a simple description of the set
$$X = \{\mathrm{disc}(R) : \ R \text{ is an order in } \mathbb{Z}[i]\ \}.$$
   (b) Is there an order of $\mathbb{Q}[\sqrt[3]{2}]$ that has discriminant $-4 \cdot \mathrm{disc}(\mathbb{Z}[\sqrt[3]{2}])$?

2. When I was a graduate student Ken Ribet asked me to determine whether or not the prime 389 divides the discriminant of a certain order T generated by an *infinite* list of explicit but hard-to-compute algebraic integers $a_2, a_3, \ldots$. Using "modular symbols" I computed that the characteristic polynomial of a_2 is
$$f = x^{20} - 3x^{19} - 29x^{18} + 91x^{17} + 338x^{16} - 1130x^{15} - 2023x^{14} + 7432x^{13} + 6558x^{12}$$
$$-28021x^{11} - 10909x^{10} + 61267x^9 + 6954x^8 - 74752x^7 + 1407x^6 + 46330x^5 - 1087x^4$$
$$-12558x^3 - 942x^2 + 960x + 148.$$
From this, we see easily that
$$\mathrm{disc}(f) = 2^{58} \cdot 5^3 \cdot 211^2 \cdot 389 \cdot 65011^2 \cdot 215517113148241 \cdot 477439237737571441.$$
Is this enough to conclude that the discriminant of T is divisible by 389? (Yes or no? Why or why not?)

3. What is the volume of the real lattice obtained by embedding the field $K(\alpha)$, for α a root of $x^3 - 4x - 2$ in $\mathbb{R}^3$ via a choice of the embedding from class (that sends α to each of the images of α in $\mathbb{R}$)? Draw a sketch of a fundamental domain for this lattice.