Graphs Intro

```
%hide
%html
<h1>An introduction to graphs in Sage</h1>
```

An introduction to graphs in Sage

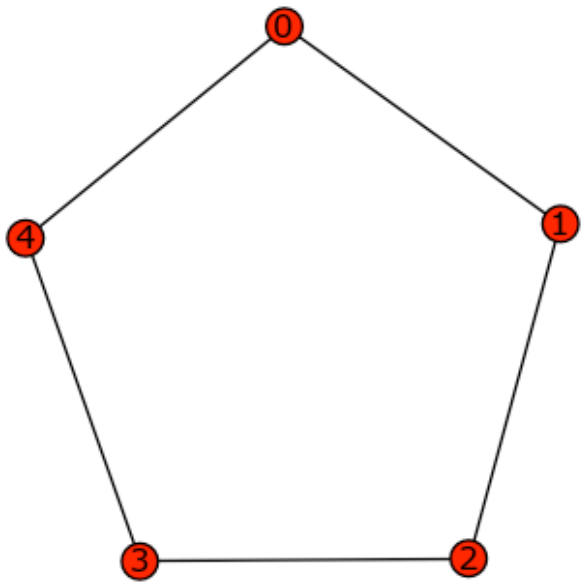
```
G = Graph()
```

```
G.num_verts()
```

```
0
```

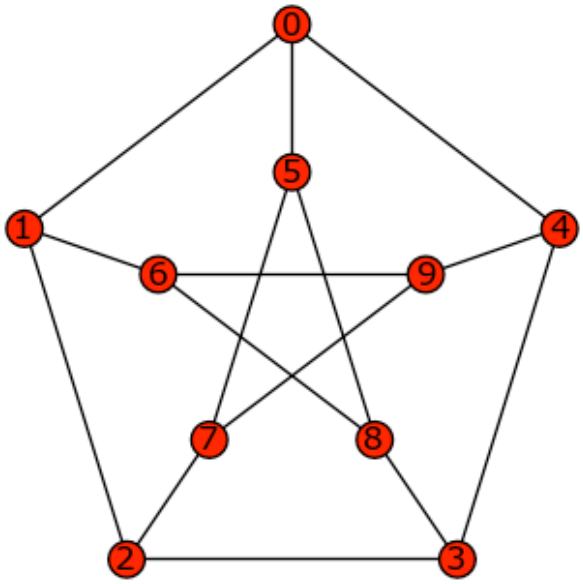
```
G.add_edges([(0,1), (1,2), (2,3), (3,4), (4,0)])
```

```
G.show()
```



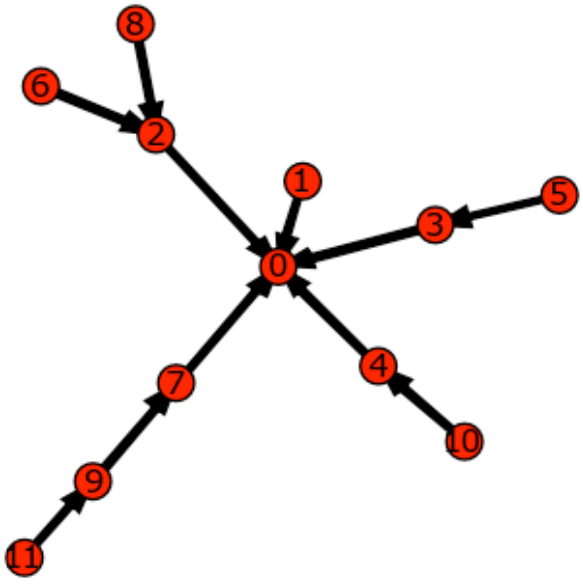

```
G = graphs.PetersenGraph()
```

```
G.show()
```




```
G = digraphs.RandomDirectedGN(12)
```

```
G.show()
```

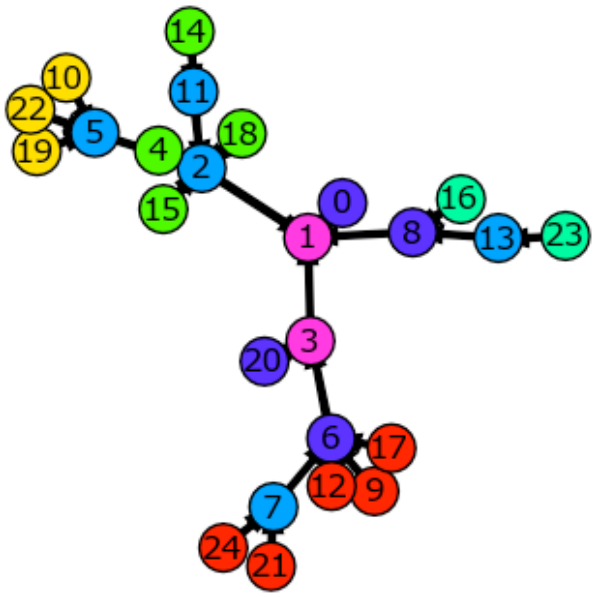


```
s = G.dig6_string(); s
```

```
'K??_?_?_?_?C?G?_?G??OA??C'
```

```
D = DiGraph('X????O???C???A????_???O???C????' + \
'O??C????C???C???O????_???C????O?@?????_??@' + \
'???G????_??@????A???C?????_??O???')
```

```
P = [  [21,24,9,12,17],[19,22,10],[15,18,14,4], \
[23,16],[7,13,5,11,2],[8,20,6,0],[1,3]  ]
D.show(partition=P, vertex_size=350)
```

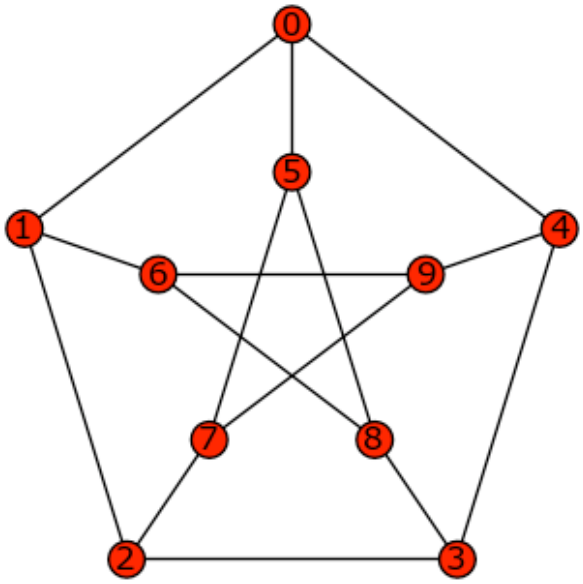



```
P = graphs.PetersenGraph()
```

```
P.adjacency_matrix()
```

```
[0 1 0 0 1 1 0 0 0 0]
[1 0 1 0 0 0 1 0 0 0]
[0 1 0 1 0 0 0 1 0 0]
[0 0 1 0 1 0 0 0 1 0]
[1 0 0 1 0 0 0 0 0 1]
[1 0 0 0 0 0 0 1 1 0]
[0 1 0 0 0 0 0 0 1 1]
[0 0 1 0 0 1 0 0 0 1]
[0 0 0 1 0 1 1 0 0 0]
[0 0 0 0 1 0 1 1 0 0]
```

```
P.show()
```




```
P.characteristic_polynomial()
```

```
x^10 - 15*x^8 + 75*x^6 - 24*x^5 - 165*x^4 + 120*x^3 + 120*x^2 -
160*x + 48
```

```
P.adjacency_matrix().charpoly()
```

```
x^10 - 15*x^8 + 75*x^6 - 24*x^5 - 165*x^4 + 120*x^3 + 120*x^2 -
160*x + 48
```

```
P.kirchhoff_matrix()
```

```
[ 3 -1  0  0 -1 -1  0  0  0  0]
[-1  3 -1  0  0  0 -1  0  0  0]
[ 0 -1  3 -1  0  0  0 -1  0  0]
[ 0  0 -1  3 -1  0  0  0 -1  0]
[-1  0  0 -1  3  0  0  0  0 -1]
[-1  0  0  0  0  3  0 -1 -1  0]
[ 0 -1  0  0  0  0  3  0 -1 -1]
[ 0  0 -1  0  0 -1  0  3  0 -1]
[ 0  0  0 -1  0 -1 -1  0  3  0]
[ 0  0  0  0 -1  0 -1 -1  0  3]
```

```
P.kirchhoff_matrix().charpoly().roots()
```

```
[(0, 1), (5, 4), (2, 5)]
```

```
G = DiGraph(class_graph(sage))
```

```
G.num_verts()
```

```
1754
```

```
time M = G.kirchhoff_matrix()
```

```
    CPU time: 21.47 s,  Wall time: 21.82 s
```

```
M = Matrix(CDF, M.dense_matrix())
```

```
M
```

```
    1754 x 1754 dense matrix over Complex Double Field
```

```
time E = M.eigenspaces()
```

```
    CPU time: 37.23 s,  Wall time: 40.24 s
```

```
max([E[i][0] for i in xrange(len(E))]) # max eigenvalues...
```

```
    4.0
```

```
[E[i][0] for i in xrange(len(E))].index(4.0)
```

```
    317
```

```
v = E[317][1].0
```

```
v.nonzero_positions()
```

```
    [416, 690, 814, 815, 816, 817, 952, 1272, 1673]
```

```
V = [G.vertices()[x] for x in v.nonzero_positions()]; V
```

```
    ['Element', 'Ideal_generic', 'MPolynomialIdeal',
    'MPolynomialIdeal_macaulay2_repr', 'MPolynomialIdeal_magma_repr',
    'MPolynomialIdeal_singular_repr', 'MonoidElement', 'SageObject',
    'object']
```

```
[G.degree(v) for v in V]
```

```
    [12, 5, 5, 1, 1, 1, 7, 80, 110]
```

```
G = graphs.PetersenGraph()
```

```
G.genus()
```

```
    1
```

```
G.breadth_first_search??
```

```
    <generator object at 0x7fe75f8>
```

```
G.girth()
```

```
    5
```

```
G.girth??
```

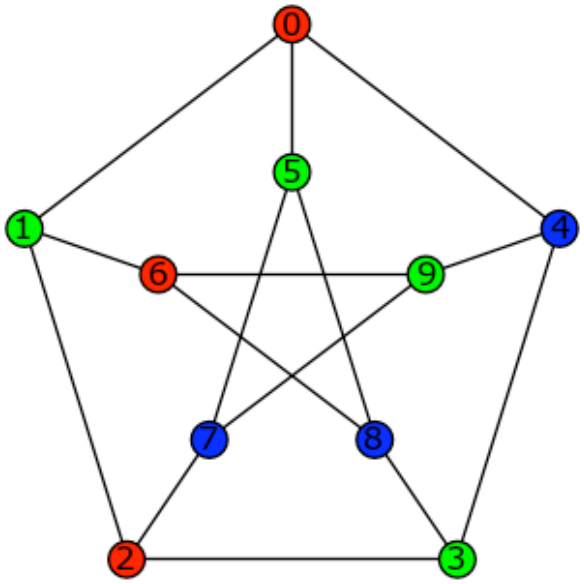
```
G.chromatic_number()
```

```
    3
```

```
C = graph_coloring.first_coloring(G, 3); C
```

```
    {'#00ff00': [1, 3, 5, 9], '#ff0000': [0, 2, 6], '#0000ff': [4, 7,
    8]}
```

```
G.show(vertex_colors=C)
```




```
f = G.chromatic_polynomial(); f
```

```
x^10 - 15*x^9 + 105*x^8 - 455*x^7 + 1353*x^6 - 2861*x^5 + 4275*x^4
4305*x^3 + 2606*x^2 - 704*x
```

```
for i in [0..4]:
   print graph_coloring.number_of_n_colorings(G, i), f(i)
```

```
0 0
0 0
0 0
120 120
12960 12960
```

```
sage.graphs.chrompoly??
```