

Midterm for Math 480A

<http://wiki.wstein.org/2008/480a>

Due Friday, May 2, 2008!

You must work alone on all these problems! You can use books, web pages on the Internet, etc., but do not email or chat people about the problems. Each problem has equal weight, and parts of problems are worth the same amount as each other. There are **4 problems**. You can email me about problems – all responses will be cc'd to **sage-uw**, so you may want to subscribe to that mailing list.

1. Which of the following programs are released under an approved open source software license that is approved by the OSI (see <http://www.opensource.org/>):
 - (a) Magma computer algebra system
 - (b) Python programming language
 - (c) Microsoft Internet Explorer web browser
 - (d) Perl programming language
 - (e) Apache web server
 - (f) Gnuplot graphics program
 - (g) Matlab math software
 - (h) Maple computer algebra system
 - (i) Firefox web browser
 - (j) Opera web browser
2. (a) Figure out how to use Sage to solve the following Sudoku puzzle.

	2			3		9		7
	1							
4		7				2		8
		5	2				9	
			1	8		7		
	4				3			
				6			7	1
	7							
9		3		2		6		5

- (b) Take the above Sudoku puzzle and put a 1 in the upper-left corner. Does the resulting puzzle have a solution or not? Prove or give a counterexample. (Your proof could involve running a calculation in Sage.)
3. Give a rough estimate (seconds, hours, months, years, millenia, or never?) about how long each of the following computations would take using Sage 3.0 on a 64-bit computer with infinite RAM. In each case, give evidence for your estimate and give valid Sage code that if it were to complete would give the required answer.
- (a) The exact integer determinant of a random 10000×10000 matrix with 1-digit integer entries. (The random numbers are uniformly distributed.)
 - (b) The prime factorization of an integer $n = pq$, where p and q are prime numbers with nearly the same number of digits, and n is a 512-bit number in binary.
 - (c) Every single decimal digit of one billion factorial: $(10^9)!$
 - (d) The rightmost last 10 decimal digits of $3^{4^5 6^7 8^9}$.
 - (e) The next prime number after 10^{10^7} .
4. Solve exactly one of the problems from the computer language benchmark game <http://shootout.alieth.debian.org/> using Sage. That game includes solutions for every single problem in Python, so to get this problem right you *must make sure* to beat whatever the corresponding timing is in Python (you can run the Python code they give to normalize the timing for your computer). It is OK to use Cython or call functions defined in any of the standard components of Sage.