Homework 3 for Math 480A

http://wiki.wstein.org/2008/480a

Due Wednesday April 23, 2008

Each problem has equal weight, and parts of problems are worth the same amount as each other. There are **5 problems**.

1. Find a bug in Sage. If you're not sure whether or not the bug you found is really a bug, write to sage-support or me `wstein@gmail.com` and confirm that it is a bug (include such email in your homework solution).

2. (a) Use Sage to draw a Cayley graph of the symmetric group S_4 of permutations of 4 objects (with respect to some choice of generators).

   (b) Explain the meaning of the graph that you just drew. Does it visually illustrate anything about the group S_4?

3. Create each of the following rings in Sage:

   (a) A ring with exactly 2008 elements.

   (b) A ring with a nonzero element a such that $a^2 = 0$.

   (c) A multivariate polynomial ring in 1000 indeterminates.

4. Give an example to show that the "field" `CC` of 53-bit precision complex numbers in Sage is *not* actually a field in the strict mathematical sense. That is, show that at least one of the axioms of a field fails. To solve this problem you must give explicit elements of `CC` and illustrate a field axiom failing for them.

5. (a) Use Sage to compute the exact number of primes that are ≤ 2008.

   (b) Show how to use Sage to compute $2008/(\log(2008) - 1)$ to 3 decimal places.

   (c) Factor the integer $2^{197} - 1$ as a product of primes. (Easy.)