Math 480b: Homework 3 -- Number Theory

Due: April 22, 2009

There are 6 problems

Problem 1: How many prime numbers are there between 1000 and 2000?

Problem 2: Create an interact with an input box for a single number n, and prints out the following information about n:

1. The ***sum*** of the prime factors of n.
2. The ***product*** of the prime factors of n.
3. The ***factorization*** of n.

Problem 3: For $n = 1, 2, ...$, let p_n be the nth prime (so $p_1 = 2, p_2 = 3, p_3 = 5$, etc.), and let g_n be the gap between p_n and p_{n+1}. Thus $g_1 = 1 = 3 - 2$ and $g_2 = 5 - 3 = 2$.

1. Compute the numbers $g_1, g_2, \ldots, g_{1000}$.
2. What is the average of the numbers $g_1, g_2, \ldots, g_{1000}$?
3. What is the most common prime gap in your list, i.e., which number appears with largest frequency among the numbers $g_1, \ldots, g_{1000}$?

4. Make a conjecture about the most common gap between primes.

Problem 4:

1. Find 10 pairs of rational numbers (x, y) such that $x^2 + y^2 = 1$, i.e., 10 rational points on the unit circle. [Hint: my lecture on Wednesday gave a formula for rational solutions to that equation.]
2. Draw a plot that illustrates the 10 rational points you found on the unit circle along with the unit circle itself.

Problem 5:

1. Compute the last 3 digits of 6^{1234} and compute in Sage **Mod(6,1000)^1234.** Notice that you can compute the last 3 digits of a big number by just computing that number modulo 1000.
2. Compute the rightmost 3 digits of $7^{123456123456123456}$. [Hint: Don't compute the power directly! You just have to compute with **Mod(7,1000)**, which is fast and easy

in Sage. You can assume that the identity that you saw in part 1 above holds in general, even though you may have never seen a proof of this fact.]

Problem 6: Can Sage compute the greatest common divisor of two 1 million digit numbers in a "reasonable amount of time" (as you define it)? [Hints: You can construct a random integer between 1 and 10 (say) using **ZZ.random_element(0,10)** command.]