1. **Getting the Riemann Spectrum as the spike values of a trigonometric series with frequencies equal to (logs of) powers of the primes:**

   To get warmed up, let's plot the positive values of the following sum of (co-)sine waves:

$$f(t) = -\frac{\log(2)}{2^{1/2}}\cos(t\log(2)) - \frac{\log(3)}{3^{1/2}}\cos(t\log(3)) - \frac{\log(2)}{4^{1/2}}\cos(t\log(4)) - \frac{\log(5)}{5^{1/2}}\cos(t\log(5))$$

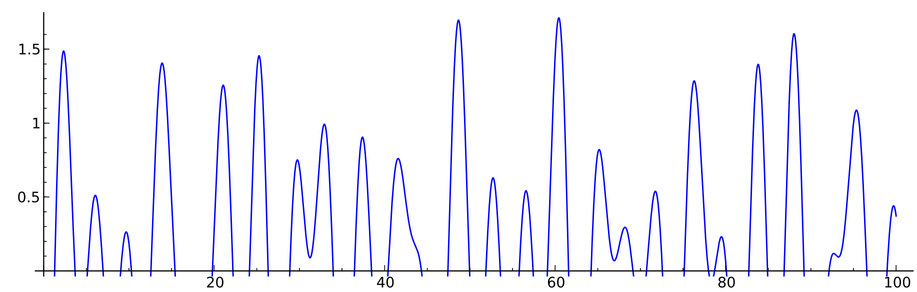

Figure 25.1: Plot of $f(t)$

Look at the peaks of this graph. There is nothing very impressive about them, you might think; but wait, for this is just a very "early " piece of an expression that consists of a sum[1] of infinitely many (co-)sine waves:

$$-\sum_{p^n}\frac{\log(p)}{p^{n/2}}\cos(t\log(p^n))$$

the summation being over all powers p^n of all prime numbers p.

Let us cut this infinite sum off taking only finitely many terms, by choosing various "cutoff values" C and forming the finite sums

$$-\sum_{p^n\le C}\frac{\log(p)}{p^{n/2}}\cos(t\log(p^n))$$

and plotting their positive values. Figures 25.2-25.5 show what we get for a few values of C.

In each of the graphs, we have indicated by red vertical arrows the real numbers that give the values of the *Riemann spectrum* that we will be discussing. These numbers at the red vertical arrows in the graphs above,

$$\theta_1, \theta_2, \theta_3, \ldots$$

[1]Here we make use of the Greek symbol $\sum$ as a shorthand way of expressing a sum of many terms. We are not requesting this sum to converge.