Math 168: Homework Assignment 6

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Due: Wednesday, Nov 9, 2005

The problems have equal point value, and multi-part problems are of the same value.

1 Problems

1. Let k be an integer, and for any function $f : \mathfrak{h}^* \to \mathbb{C}$ and $\gamma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{GL}_2(\mathbb{Q})$, set $f|[\gamma]_k(z) = (cz+d)^{-k} f(\gamma(z))$. Prove that if $\gamma_1, \gamma_2 \in \mathrm{SL}_2(\mathbb{Z})$, then for all $z \in \mathfrak{h}$ we have
$$f|[\gamma_1\gamma_2]_k(z) = ((f|[\gamma_1]_k)|[\gamma_2]_k)(z).$$
(I mostly did this on the blackboard in class.)

2. Prove that for any $\alpha, \beta \in \mathbb{P}^1(\mathbb{Q})$, there exists $\gamma \in \mathrm{SL}_2(\mathbb{Z})$ such that $\gamma(\alpha) = \beta$.

3. By hand write down the coefficients of 1, q, q^2, and q^3 of the Eisenstein series E_8.

4. Explicitly compute the Victor Miller basis for $M_{28}(\mathrm{SL}_2(\mathbb{Z}))$.

5. Let $f = E_4 E_6 \Delta$.
   (a) What is the weight of f?
   (b) Determine the location of all the zeros of f.