

Spring 2010

# Math 480b Project: Laminate Materials Stress and Failure Calculations Using Sage

Group Members:

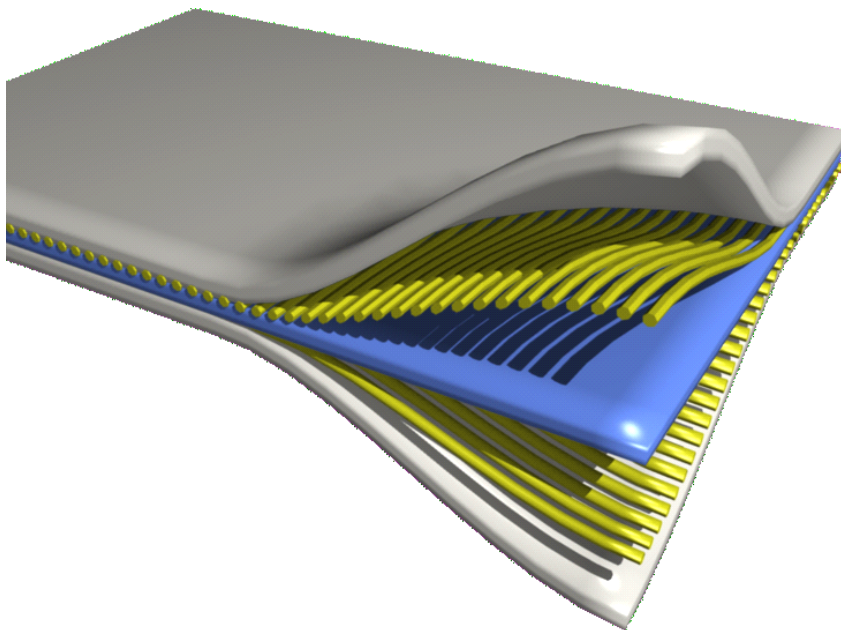
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In engineering, an important material widely used in modern fabrication is composite laminates. A laminate of this kind consists of several layers of different fibrous materials held together by some adhesive, each of which has a single directionality. Generally, neighboring plies vary in the direction of materials, in order to increase the strength of the overall composite. Each ply is generally quite thin, allowing for the properties of the composite material to be finely controlled. Modern engineering demands, such as stiffness, strength, and coefficient of thermal expansion can therefore often be met to a greater extent than with other materials.



**Figure 1: Composite laminate materials consist of layers of fibers in sheets of resin, bound together with layers of adhesive. Varying the direction of the fibers in each layer increases the strength of the composite material.<sup>1</sup>**

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<sup>1</sup> Public domain image: [http://upload.wikimedia.org/wikipedia/commons/1/13/Composite\\_3d.png](http://upload.wikimedia.org/wikipedia/commons/1/13/Composite_3d.png)

Composite laminates are widely used in modern industry: for example, a Boeing 787 is constructed of up to 50% composite laminates (Figure 2). An understanding of the behavior of a wide variety of laminates, and the ability to model very specific composites, is necessary to ensure that materials used in industry meet performance and safety requirements.

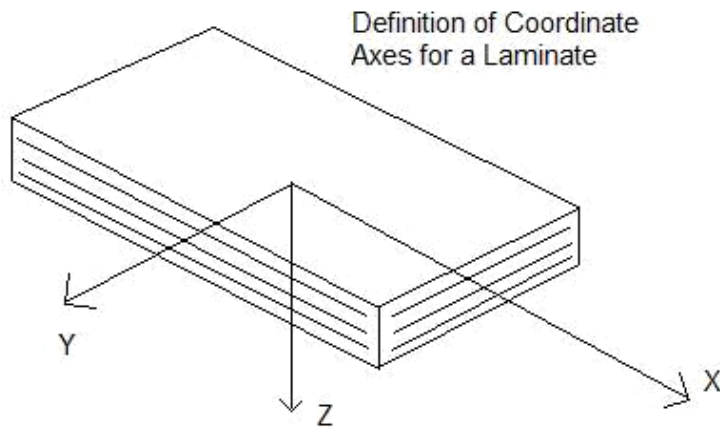


**Figure 2: A Boeing 787 is constructed of up to 50% composite materials<sup>2</sup>**

Since the main utility of composite laminates involves achieving particular physical properties, it is important that the potential failure points of these materials be determined. Particularly, we are interested in modeling the ways in which the material bends (distortion), how the load affects each ply (stress conditions), and where and how the material might break (failure conditions). Unlike homogeneous materials, which are relatively easy to model, composite materials require a more complex approach. The input required to calculate these stresses and strains is complex, and the modeling must be done in three dimensions (Figure 3).

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<sup>2</sup>public domain image [http://en.wikipedia.org/wiki/File:Boeing\\_787first\\_flight.jpg](http://en.wikipedia.org/wiki/File:Boeing_787first_flight.jpg)



**Figure 3: Composite laminates must be modeled in 3D to calculate stress and strain (Taken from the attached reference document, “A Summary of Classical Lamination Theory”).**

First, the appropriate forces must be input: the vertical and horizontal stress forces, shear forces, and moment (twisting) forces. Additionally, the material properties are required for each ply: thermal expansion, moisture expansion, elasticity, and the angle of the fiber in each layer.

Finally, the environmental conditions—ambient temperature and moistures—must be included in the model. The calculations necessary to derive the aforementioned output properties involve calculating products of 6x6 matrices, and if done by hand, require several hours to compute.

Therefore, a computerized method for performing these calculations and producing the desired output is a far more attractive option.

The Sage-based modeling program uses the @interact feature to request user input for material properties, environmental conditions, and forces for the material to be analyzed. It then calculates a reduced stiffness matrix for each component type in the laminate specified, transforms these matrices, and uses them to calculate the ABD matrix. The ABD matrix is the

central component of the calculation, because it defines the relationship between the loads and strains in the laminate. For each component type in the laminate, a corresponding ABD matrix is derived, assembled, and inverted. The thermal and moisture expansion coefficients for each ply are then calculated and combined with the transformed reduced stiffness matrices to determine the thermal and moisture resultants. At this point, the program has all the information it needs to derive the midplane stresses and strains, as a product of the ABD matrices and the thermal/moisture resultants. Finally, the individual ply strains are calculated from the midplane stress and strain results, and the ply stresses are derived from the product of the reduced stiffness matrix and the difference between the strain and the thermal/moisture components. (For further reference, see the attached document, "A Summary of Classical Lamination Theory".)

The output results are displayed in a text table, with each quantity clearly annotated. If the material fails at any point, a message is displayed indicating the particular ply that breaks. From this, any desired composite laminate material could be analyzed to quickly obtain any of the listed outputs. The program is capable of handling even large input values, i.e, extremely high forces. Future work on the program might involve adding graphical displays, such as bar charts, to show the relative stress and strain on each ply in the composite in question.